Experimental back-scattering data for a coefficient identification problem

Nguyen Trung Thành[a,*], Larisa Beilina[b], Michael V. Klibanov[a] and Michael A. Fiddy[c]

[a]Department of Mathematics & Statistics,
University of North Carolina at Charlotte, Charlotte NC 28213, USA.
[b]Department of Mathematical Sciences,
Chalmers University of Technology and Gothenburg University,
SE-42196 Gothenburg, Sweden
[c]Optoelectronics Center,
University of North Carolina at Charlotte, Charlotte NC 28213, USA.
[*]Presenter, email: tnguy152@uncc.edu

Consider the wave equation

$$\epsilon^2(x)u_{tt}(x,t) - \Delta u(x,t) = 0, x \in \mathbb{R}^3, t \in (0,\infty) \tag{1}$$

with the initial conditions

$$u(x,0) = 0, \quad u_t(x,0) = \delta(x - x0), x \in \mathbb{R}^3. \tag{2}$$

In electromagnetic wave propagation theory, the coefficient ϵ represents the refractive index of the medium in which the wave propagates. Assume that the $\epsilon(x) \in [1, 1+d]$ with some positive constant d. Moreover, $\epsilon(x) = 1$ outside a bounded domain $\Omega \subset \mathbb{R}^3$ and $x_0 \notin \Omega$.

The inverse problem to be considered in this talk is to reconstruct the refractive index $\epsilon(x), x \in \Omega$ using one boundary measurement

$$u(x,t) = g(x,t), (x,t) \in \Gamma \times (0,\infty), \tag{3}$$

where Γ is the part of the boundary $\partial\Omega$ corresponding to the back scattering of the incident wave by the inhomogeneous medium.

This inverse problem is well-known to be ill-posed. Conventional gradient-based optimization methods usually require a good first guess of the true coefficient due to the presence of several local minima of the objective functional. However, this requirement is not always possible in many practical situations. To avoid this requirement, we make use of a globally convergent method recently proposed by Beilina and Klibanov (for this method, see L. Beilina & M. Klibanov *Approximate Global Convergence and Adaptivity for Coefficient Inverse Problems*, Springer, 2012). This method has been proved to be globally convergent in the context of some mathematical models. That means, we can obtain good reconstructions without the knowledge of a good initial guess.

In this talk, we discuss challenges of using experimental data in this inverse problem. We show a huge misfit between experimental data and simulated one and present data processing steps for converting the former to an approximate of the latter.

We also show reconstruction results obtained by the globally convergent method.

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